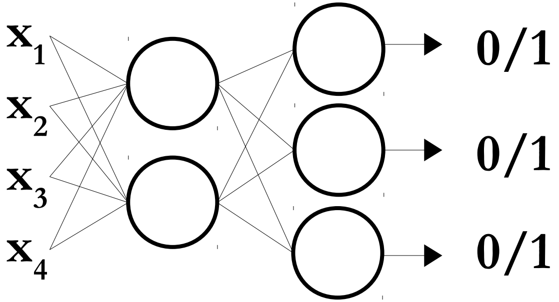
Deep Learning

How does the algorithm make a decision?

mathematical conceptual

Feedforward



$$H_{in} = XW_1$$

$$H_{out} = step(H_{in})$$

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How do you determine the right parameters for the algorithm?

mathematical conceptual

Cost function/Loss function:

$$\text{Mean Squared Error} = \frac{1}{N}\sum_{e}\sum_{n}(O_{out\ e,n} - Y_{e,n})^2$$

$$\textbf{MSE} = \frac{1}{N}\sum_{e}\sum_{n}(step(step(XW_1)W_2)_{e,n} - Y_{e,n})^2$$

Gradient Descent:

$$W_i := W_i - \alpha \cdot \frac{\partial\, MSE}{\partial\, W_i}(W_i)$$

$$\frac{\partial\, \textbf{MSE}}{\partial\, \textbf{W}_1}(\textbf{W}_1) =$$

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N: e * n

Chain Rule

$y = 3x - 2$

$z = y^2$

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$$\frac{dz}{dx} =$$

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$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

Chain Rule

x = 5v - 2

y = 3x - 2

$z = y^2$

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$x = 5v - 2$

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$y = 3x - 2$

$z = y^2$

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

Chain Rule

$y = 3x - 2 \qquad \frac{dy}{dx} = 3$

$z = y^2 \qquad \frac{dz}{dy} = 2y$

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

Chain Rule

x = 2:

$y = 3x - 2$ $\frac{dy}{dx} = 3$

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Chain Rule

x = 2:

$y = 3x - 2$ $\qquad \frac{dy}{dx} = 3$ $\qquad y(2) = 4$

$z = y^2$ $\qquad \frac{dz}{dy} = 2y$ $\qquad z(4) = 16$

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

Chain Rule

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$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$\frac{dz}{dx}(2) = 8 * 3$$

$$= 24$$

Chain Rule

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x = 2:

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$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

x = 2:

$z = 9x^2 - 12x + 4$

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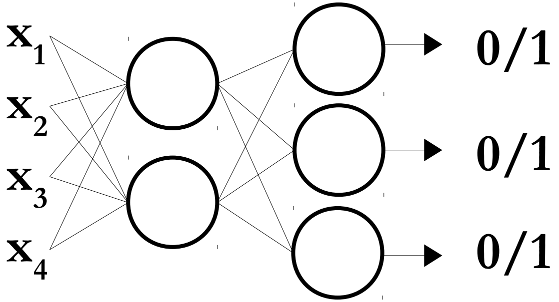

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N: e * n

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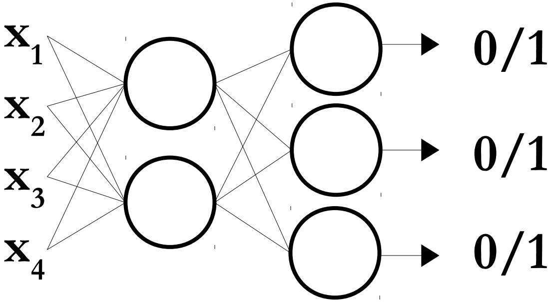

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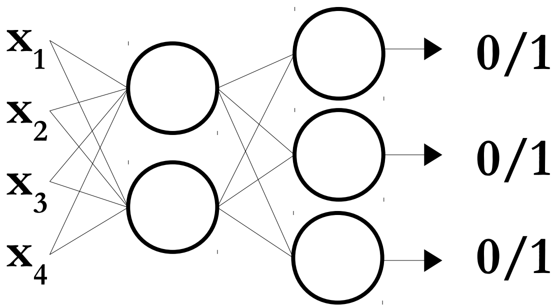

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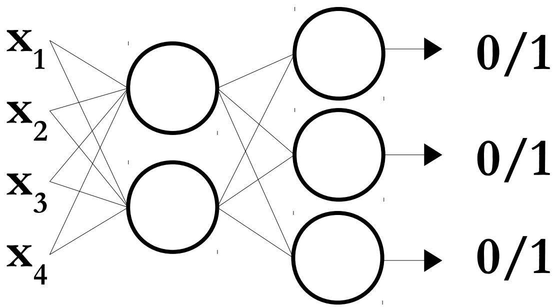

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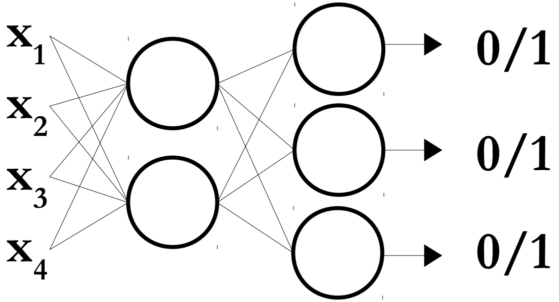

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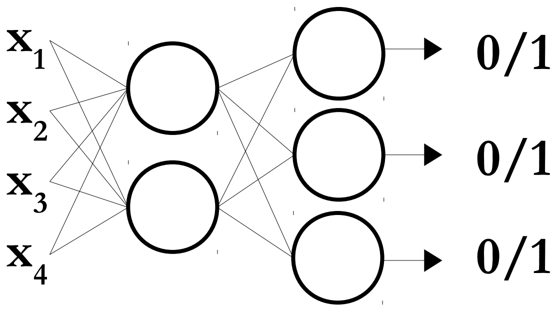

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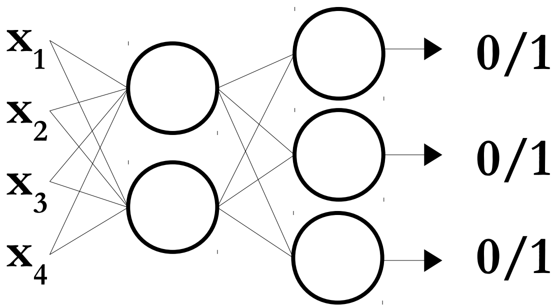

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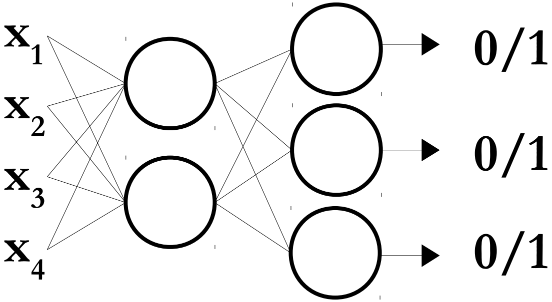

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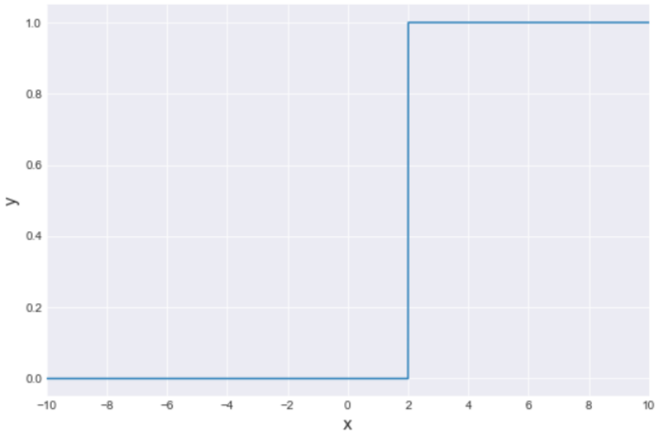
N: e * n

Activation Function

Step Function

$$y = \begin{cases} 0 & \text{if } x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$$



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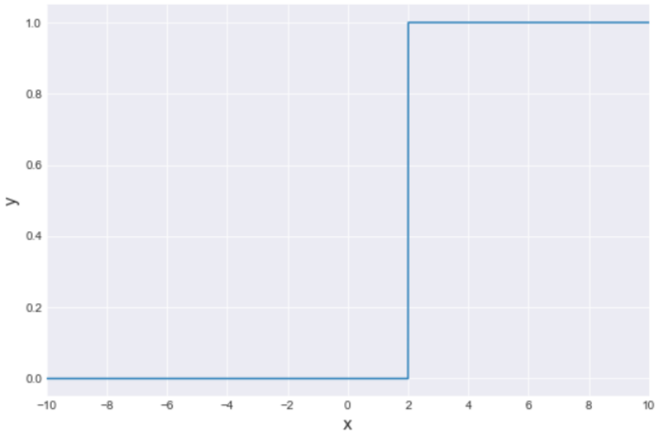

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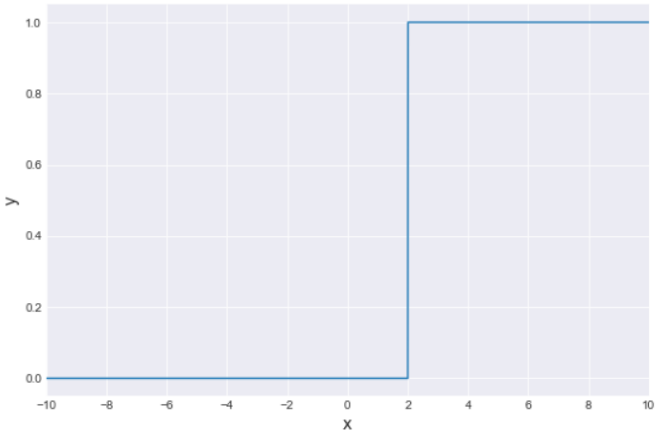
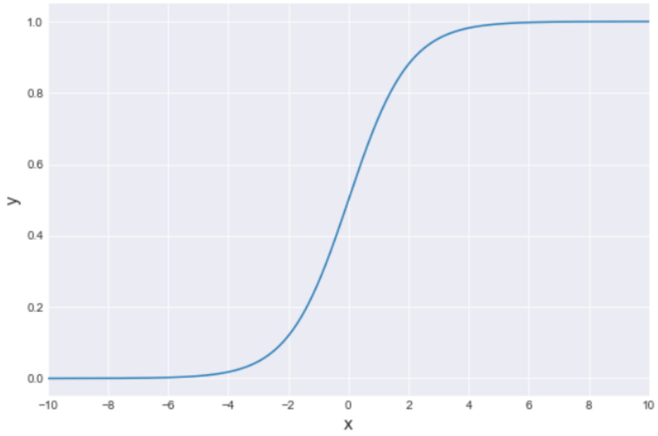


Sigmoid Function

$$y = \frac{1}{1 + e^{-x}}$$



Activation Function

Step Function

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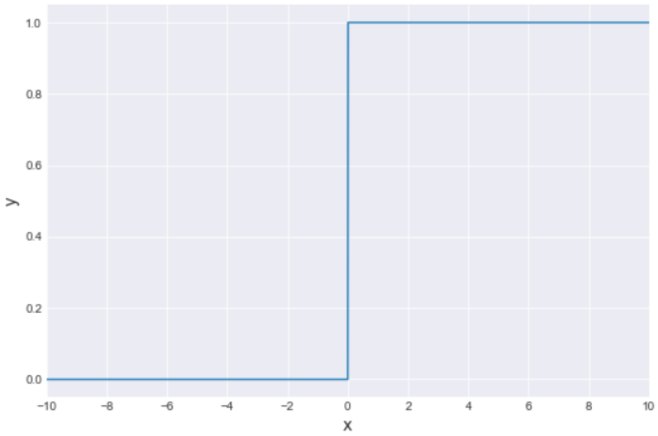

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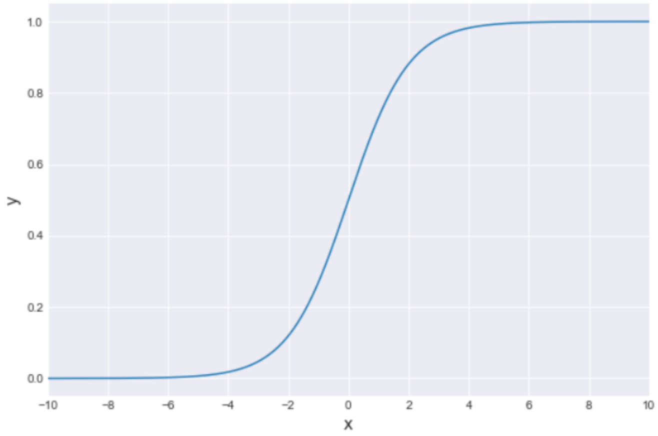

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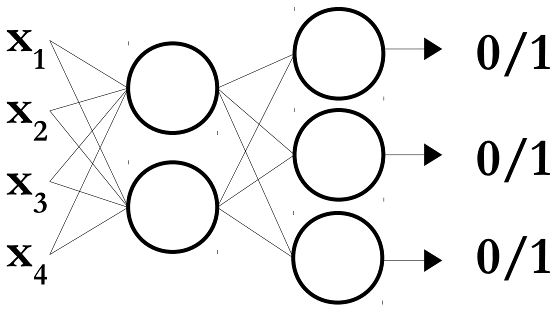

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N: e * n

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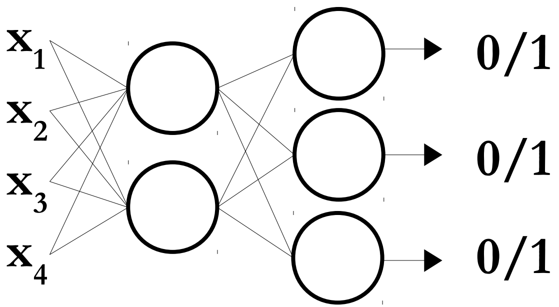

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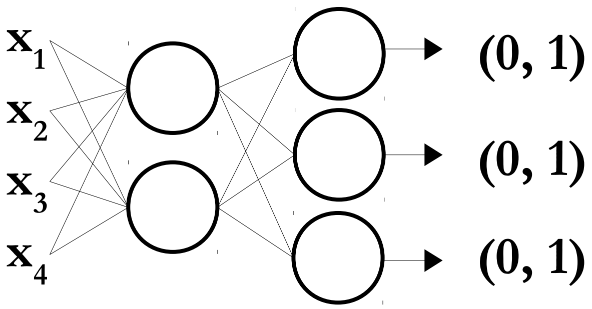

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